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RADAR SCATTERING: A TIME-FREQUENCY PERSPECTIVE

H. N. Kritikos
University of Pennsylvania
Dept. of Electrical Engineering.
200 S. 33 Str. E-Mail: kritikos@ee.upenn.edu Tel: (215) 989-8112 Fax: (215) 573-2068
Philadelphia, PA 19104 -USA

J. G. Teti, Jr.
Lambda Science, Inc.
P.O. 238 E-Mail: drteti@lamsci.com Tel: (215) 957-5354 Fax: (215) 957-5494
Wayne, PA 19087 -USA

Abstract

Time-frequency analysis is used to study electromagnetic scattering from canonical objects leading to the detection, localization and extraction of response features that result from the object's physical attributes. The time-frequency analysis techniques are motivated by signal representation in phase space, and are formulated using the Bargman transform. Bargman transform techniques allow the phase space to be parametrically represented in terms of complex polynomials, and elliptical filters can be constructed to crop selected areas of the phase plane. The filters are applied to the scattering from spheres and thin wires, and the prominent scattering features are identified and extracted.

1.Background

A number of investigators [1,2,3,4,5] have introduced a form of the Bargman transform to analyze signals in phase space. The Bargman transform is defined as

$$\mathrm{Bf}(z)=\frac{1}{\pi^{1/4}}e^{-z^2/4}\int_{-\infty}^{\infty}e^{zx-x^2/2}f(x)dx, \qquad 1$$

and is an isometry from the space of square integrable functions $L^2(\Re)$ to the space $L^2\left(\mathbb{C},e^{-|z|^2/4}dz\right)$ which is known as the Fock space $\mathcal{F}$. The Fock space is defined as

$$\mathcal{F}=\left\{F: F \text{ is an entire function on } \mathbb{C},\ \|F\|_{\mathcal{F}}^2=\int_{-\infty}^{\infty}|F(z)|e^{-|z|^2/4}dz<\infty\right\} \qquad 2$$

where $z = q - ip$; q and p are the phase space coordinates. The Fock Space is the space of entire analytic functions defined in the whole complex plane $\mathbb{C}$.

The basic notion of the Bargman transform comes from physics where the parameters q and p are known as the spatial coordinate and momentum of a system. Describing a dynamical system in terms of its spatial coordinate q and momentum p is known as the phase space representation. In traditional form, the Bargman transform can be expressed in terms of the phase space variables p and q as

$$Bf(z)=\frac{1}{\pi^{1/4}}e^{-\left(p^2+q^2\right)/4}\int_{-\infty}^{\infty}e^{-ipx-(x-q)^2/2}f(x)dx. \qquad 3$$

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2. Hermite Function Expansions

An important analytical tool in the representation of signals is their expansion in terms of Hermite functions $\varphi_n(x)$ given by

$$\varphi_n(x) = \frac{e^{-\frac{x^2}{2}}}{\pi^{1/4}} \frac{H_n(x)}{\sqrt{2^n n!}} \text{ with Bargman transform } \varsigma_n(z) = \frac{z^n}{\sqrt{2^n n!}} \qquad 4$$

where $H_n(x)$ is the Hermite Polynomial.

The expansion of the target returns in terms of the orthogonal Hermite functions enables one to apply elliptical filter disks in phase space which extract the signature within the disk. Recognizing that the filtering process is a trace class operator, a spherical filter disk can be implemented by finding a Hermite function expansion of the signal in the x domain, and weighting the coefficients with the eigenvalues of the filter operator $\mathbf{P}_{Sk}$ as follows

$$\mathbf{P}_{SR} f(x) = \sum_{n=0}^{R^2/2} \lambda_n \langle \varphi_n(x), f(x) \rangle \varphi_n(x), \qquad 5$$

R is the radius of the filter disk in the phase plane, λ_n is the eigenvalue $\lambda_n = \frac{1}{n!}\gamma(n+1, R^2/2)$, and γ is the incomplete gamma function.

The Hermite expansion is very robust and easy to implement numerically. The corresponding monomials are also orthogonal in phase space, and provide a convenient basis for expanding signals. Furthermore, the monomials ideally exhibit the analytical properties of the transformed signals.

3. Applications

Two examples have been chosen to demonstrate the use of the Bargman transform for filtering. One is the sphere for which the exact solution is known in terms of the Mie series [6]. The other is the thin wire where we used the Ufimtsev approximate solution [6]. These two examples represent two extremes. The return signal from the sphere is mainly due to body scattering, whereas the return signal from the thin wire is due mainly to resonant scattering.

An example of filtering the scattering response of a perfectly conducting sphere is shown in Figure I. A phase space filter is illustrated where most of the energy is captured with R=15. The time domain reconstruction shows a reconstructed signal that is typical for a conducting sphere.

Scattering from the wire is shown in Figure II where only a portion of the phase plane was cropped. This was done because the signal is more spread out in the time axis and a full reconstruction will require a very large R. A partial reconstruction using R=8 shows that the main features of the selected region are well approximated.

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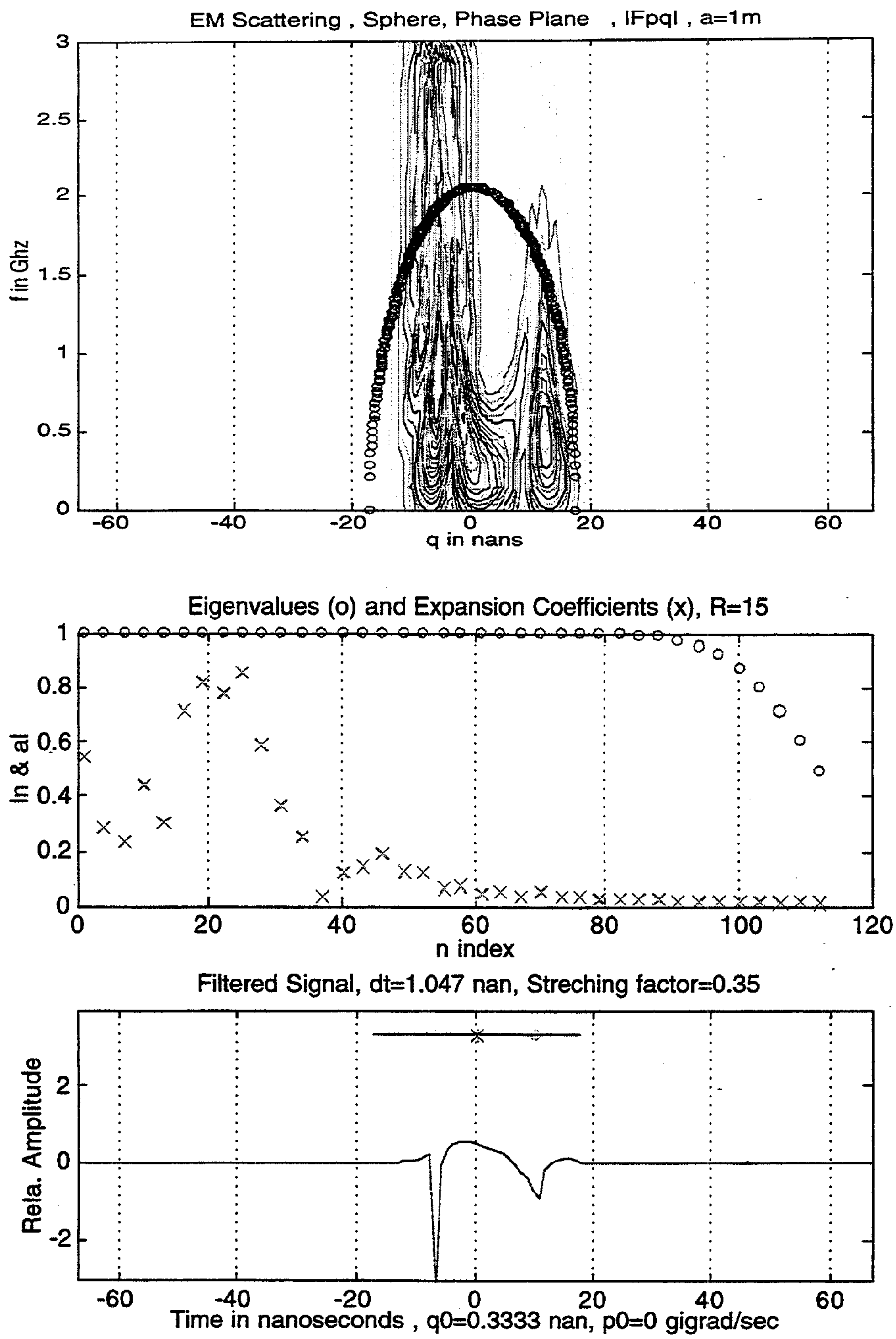

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Figure I. Scaterring from a perfectly conducting Sphere. Radius =1 m. Filter Radius =10.

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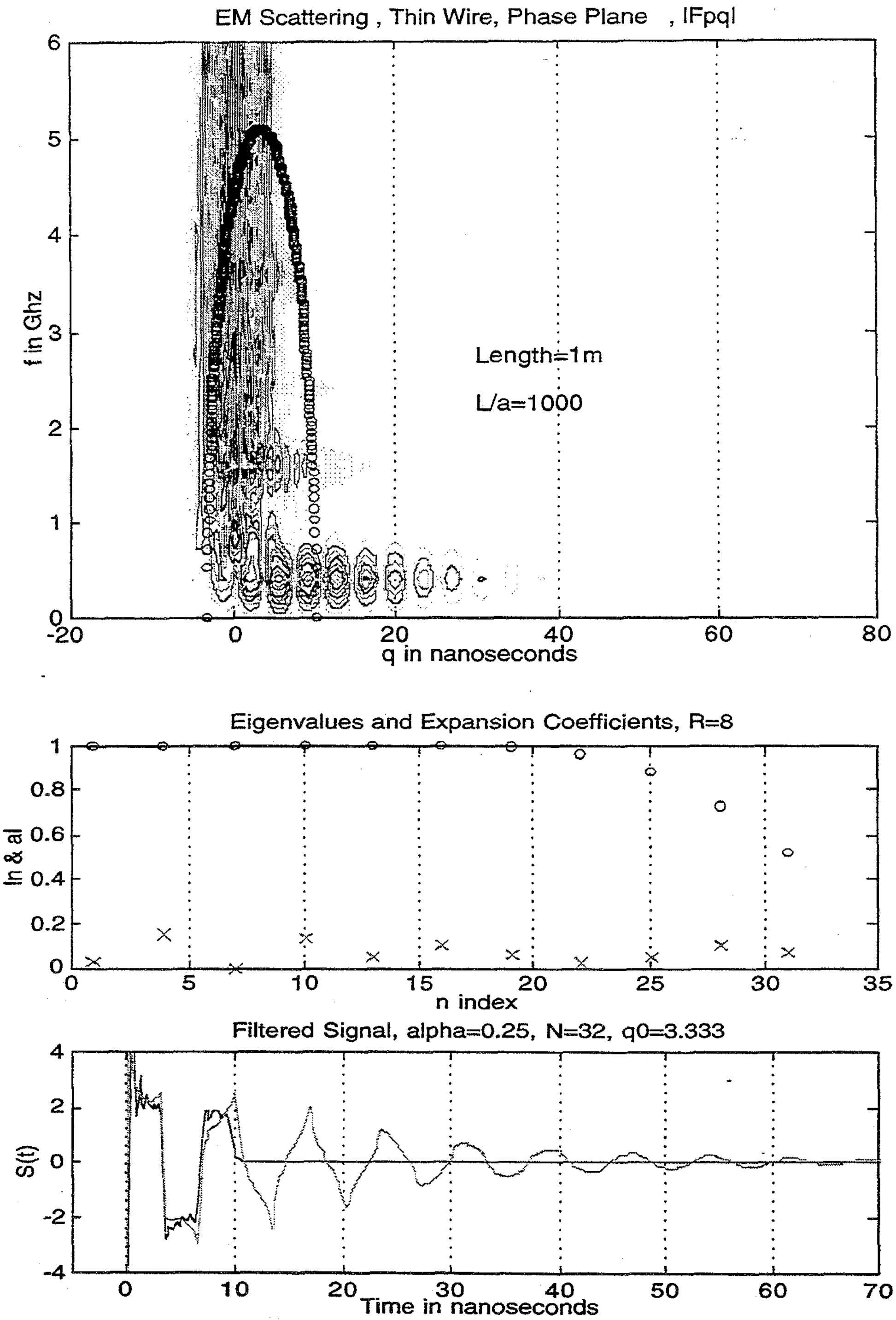

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FigureII . Scattering from thin Wire. L=1m. L/a=1000. Angle of Incidence is 90°. R=8.

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4. Conclusions

The Bargman transform allows the representation of signals in phase space where both the time and frequency features can be displayed simultaneously. Distinguishing features in the phase space can be extracted by placing elliptical filters in the selected areas. For the special case of elliptical filters, the filtering can take place in the time domain by using the proper Hermite function expansions. The ellipticity and the placement of the filters can be chosen to accentuate the temporal response of targets like the sphere or the frequency response of targets like that of the ringing wire. The placement of the filters is arbitrary and can be positioned any place in phase space. The advantage of the suggested method is that it has the potential to be performed in the time domain without having to transform into phase space, crop, and then transform back into the time domain.

5. References

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